Neutral depletion versus repletion due to ionization[a)]

A. Fruchtman,[1,b)] G. Makrinich,[1] J.-L. Raimbault,[2] L. Liard,[2] J.-M. Rax,[2] and P. Chabert[2]
[1]*H.I.T.—Holon Institute of Technology, 52 Golomb St., Holon 58102, Israel*
[2]*Laboratoire de Physique et Technologie des Plasmas, Ecole Polytechnique, 91128 Palaiseau, France*

Recent theoretical analyses which predicted unexpected effects of neutral depletion in both collisional and collisionless plasmas are reviewed. We focus on the depletion of collisionless neutrals induced by strong ionization of a collisionless plasma and contrast this depletion with the effect of strong ionization on thermalized neutrals. The collisionless plasma is analyzed employing a kinetic description. The collisionless neutrals and the plasma are coupled through volume ionization and wall recombination only. The profiles of density and pressure both of the plasma and of the neutral-gas and the profile of the ionization rate are calculated. It is shown that for collisionless neutrals the ionization results in neutral depletion, while when neutrals are thermalized the ionization induces a maximal neutral-density at the discharge center, which we call neutral repletion. The difference between the two cases stems from the relation between the neutral density and pressure. The pressure of the collisionless neutral-gas turns out to be maximal where its density is minimal, in contrast to the case of a thermalized neutral gas. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2844374]

I. INTRODUCTION

Space and laboratory plasmas can be significantly affected by neutral depletion. The effect of neutral depletion in gas discharges has already been addressed in some early insightful studies.[1–4] However, only in recent years, with the growing use of lower pressure and higher power radio-frequency discharges, is the importance of neutral depletion becoming fully recognized.[5–18] Decrease of the neutral density,[5–7,9,12,14,16–18] relaxation oscillations,[8] and neutral-gas heating[10,11,13,15,18] have been measured.

Recently we investigated theoretically various aspects of neutral depletion in gas discharges.[19–25] We have examined the plasma and neutral steady-state in collisional and collisionless plasmas (meaning that the ions are collisionless). In the case of such a collisionless plasma we found that as a result of intense ionization the density profile of collisionless neutrals, as described in Ref. 24, is very different than that of thermalized neutrals, described in Ref. 22, exhibiting depletion versus repletion. In Ref. 24 we identified the source of the different neutral density profiles as the different relations between density and pressure for the collisionless and thermalized neutrals. The two different neutrals gases in Ref. 22 and in Ref. 24 were assumed to interact both with a collisionless plasma. However, that same collisionless plasma was analyzed with a kinetic model in Ref. 22 and with a fluid picture in Ref. 24. Here we reanalyze this interaction of the collisionless plasma with collisionless neutrals, this time employing a kinetic model for the ions, as we did in Ref. 22 where neutrals were thermalized. This allows us to compare the interaction of neutrals of the two different dynamics, collisionless versus thermalized, with a collisionless plasma that is described by the same (kinetic) model. We calculate in

detail the profiles of the plasma and neutral flows (for a case that we found to be first addressed in Ref. 2) and discuss in particular the above mentioned relations between the neutral density and pressure, explaining the source of depletion and repletion.

In Sec. II we briefly review our recent theoretical studies. The models for the plasma and for the neutrals are presented in Sec. III and in Sec. IV, respectively. In Sec. V the relation between the neutral density and pressure and the resulting depletion or repletion are discussed. In Sec. VI asymptotic results and numerical examples are presented.

II. OUR RECENT THEORETICAL STUDIES OF NEUTRAL DEPLETION

In weakly ionized plasmas, in which the neutral density and temperature are uniform (and nonlinear processes, such as two-step ionization, are small), particle balance is decoupled from power balance and the electron temperature is found to be related to a single similarity variable, the product of the neutral gas pressure, and the plasma spatial extent (the Paschen parameter).[26–31] The plasma density is determined by power balance and increases monotonically with deposited energy, as does the plasma particle flux. When ionization is intense and neutral density is significantly modified, so that neutral pressure is not uniform, the above-mentioned similarity variable is no longer well defined. Moreover, particle balance and power balance become coupled and so do ionization and transport.

In our recent theoretical studies[19–25] we have investigated various aspects of neutral depletion in gas discharges and have examined the plasma and neutral steady-state in both collisional and collisionless plasmas. In all these studies the total number of neutrals has been found to replace the Paschen parameter as the similarity variable that determines the electron temperature. In Ref. 19 we examined a colli-

---

15, 057102-1

sional plasma in the Schottky regime (a regime described in Ref. 26) that is in pressure balance with the neutral gas. We derived and solved analytically a governing nonlinear equation, transforming it into the algebraic Kepler equation.[19] We have shown in Ref. 19 that because of the inherent coupling of ionization and transport, an increase of the energy invested in ionization can nonlinearly enhance the transport process. Such an enhancement of the plasma transport due to neutral depletion was shown to result in an unexpected decrease of the plasma density when power is increased; despite the increase of the flux of generated plasma.[19] The neutral depletion in a collisional plasma has been described further in Ref. 22. The Schottky regime has been described in a greater detail and neutral depletion in the Godyak regime (a regime described in Ref. 28) has also been formulated.

In Refs. 23 and 25 we studied neutral depletion due to neutral-gas heating by the plasma. In Ref. 23 an energy equation was derived in which neutral heating through collisions with plasma electrons is balanced by heat conduction to the boundaries. In Ref. 25 it was shown that the fraction of power deposited in the neutrals does not depend on the amount of deposited power or on the neutral density profile but is rather determined by the atomic cross sections and by the electron temperature only. Asymptotic relations for a small coefficient of neutral heat conductivity have been found. It is shown there that a low heat conductivity of the neutral gas is followed by a high neutral temperature that results in a high neutral depletion even if the plasma pressure is small.

In Refs. 20–22 and 24 we addressed the neutral depletion in a collisionless plasma. In Refs. 20, 21, and 24 the neutral gas was assumed collisionless so that the neutral-gas atoms move ballistically and, contrary to the collisional case in Refs. 19, 23, and 25, do not exchange momentum with the ions, rather they interact with the ions only through volume ionization and wall recombination. The dynamics of neutrals with such assumptions has also been analyzed in past studies in Refs. 1–4, 9, 12, and 14. We performed an analysis of the steady-state both for a closed system and for an open system with a net mass flow. We have shown that in all these cases the neutrals are depleted as a result of the ionization and that, as expected, the neutral density decreases in the direction of the neutral flow. The analysis describes what is called ion pumping.[6–8] The analysis in Ref. 20 and in Ref. 24 for an open system with a net mass flow has relevance to plasma sources such as helicons that are being considered for use as thrusters.[32–41] We have shown that, because of the presence of the sheath, the power flow into the backwall is larger than the power flow along the plasma exiting the source. The calculated profiles of the plasma and neutral particles along the channel were used to derive expressions for the thrust, propellant utilization, specific impulse, and efficiency of a plasma source used as a thruster. The energy cost for ionization and the backwall energy losses were shown to significantly reduce the efficiency.[24]

In Ref. 25 we have discussed what we called neutral pumping, which we distinguish from ion pumping. Neutrals are accelerated towards the boundaries through charge-

exchange collisions with fast ions. The fast neutrals resulting from the charge-exchange collisions are assumed to quickly leave the system. We note that the term "neutral pumping" is sometimes used to describe what we called above ion pumping. We suggested in Ref. 25 that a thruster could employ a collisional plasma where thrust is delivered mostly through pumped neutrals rather than through pumped ions.

An unexpected dynamics of the neutrals has been revealed in Ref. 22 where neutrals were assumed to be thermalized rather than to move ballistically. Strong ionization resulted in this case in a maximum of the neutral-gas density surprisingly located at the center of the discharge. We call this density increase of the neutrals in the direction of their flow a neutral repletion. The source of the difference between the density profiles of the collisionless neutrals[21,24] and of thermalized neutrals[22] has been unfolded in Ref. 24 and is explained again in Sec. V of this paper.

In this paper we reanalyze the case studied in Ref. 24, this time with the collisionless ions described kinetically. In the next section we present the kinetic model for the plasma.

III. THE COLLISIONLESS PLASMA

In our recent analysis of the interaction between a collisionless plasma and thermalized neutrals[22] we used a kinetic model[42–45] for the plasma. For redescribing the interaction between a collisionless plasma and ballistic neutrals we use here this kinetic model for the plasma instead of the fluid model that we used in Refs. 21 and 24.

We assume a collisionless plasma in which volume ionization is balanced by loss at the boundaries. The collisionless plasma is assumed quasi-neutral and one-dimensional, where all variables vary along z only. The continuity equation is

$$\frac{d\Gamma}{dz} = g(z), \tag{1}$$

where Γ is the plasma particle flux-density and g is the ionization rate. We express the ion density n_i and the electron density n_e as

$$n_i(z) = \left(\frac{m}{2e}\right)^{1/2} \int_0^z \frac{g(z')dz'}{[\varphi(z') - \varphi(z)]^{1/2}}, \tag{2}$$

and

$$n_e(z) = n_0 \exp\left[\frac{e\varphi(z)}{T}\right]. \tag{3}$$

Here φ is the electrostatic potential, T the electron temperature, m the ion mass, and e is the elementary charge. The peak of the ion and electron densities is at $z=0$ and we choose $\varphi(0)=0$. Assuming quasineutrality, $n \equiv n_i = n_e$, so that $n(0)=n_0$, we combine Eqs. (2) and (3) to

$$n_0 \exp\left[\frac{e\varphi(z)}{T}\right] = \left(\frac{m}{2e}\right)^{1/2} \int_0^z \frac{g(z')dz'}{[\varphi(z') - \varphi(z)]^{1/2}}. \tag{4}$$

As is usually done, we solve this integral equation for $\varphi(z)$ by employing Abel inversion, multiplying the equation by

$(\varphi - \varphi_1)^{-1/2}$ and integrating over φ from zero to φ_1. The result is

$$
n_0 \exp\left(\frac{e\varphi_1}{T}\right) \int_0^{(-\varphi_1)^{1/2}} 2ds \exp\left(\frac{es^2}{T}\right)
$$

$$
= \pi \left(\frac{m}{2e}\right)^{1/2} \int_0^{\varphi_1} d\varphi' g(\varphi') \frac{dz'}{d\varphi'}. \tag{5}
$$

This last relation can be written as

$$
\Gamma(z) = \int_0^z dz' g(z') = \Gamma(\psi)
$$

$$
= \int_0^{\psi} d\psi' g(\psi') \frac{dz'}{d\psi'} = \frac{2\sqrt{2}}{\pi} n_0 c F(\psi^{1/2}), \tag{6}
$$

where F is the Dawson integral,

$$
F(x) \equiv \exp(-x^2) \int_0^x \exp(t^2) dt = \exp(-x^2) \frac{\sqrt{\pi}}{2} \operatorname{erf} i(x), \tag{7}
$$

and

$$
\psi \equiv -\frac{e\varphi}{T} \geq 0, \quad c \equiv \sqrt{\frac{T}{m}}. \tag{8}
$$

From Eq. (6) we obtain

$$
\frac{d\Gamma(\psi)}{d\psi} = \frac{2\sqrt{2}}{\pi} n_0 c \left[-F(\psi^{1/2}) + \frac{1}{2\psi^{1/2}} \right] = g(\psi) \frac{dz}{d\psi}. \tag{9}
$$

The electric field is infinite at the plasma boundary where the square brackets vanish. This happens at

$$
\psi = \psi_w = 0.854. \tag{10}
$$

The maximal plasma particle flux-density is therefore

$$
\Gamma_{\max} = \Gamma(\psi_w) = \frac{2\sqrt{2}}{\pi} n_0 c F(\psi_w^{1/2}) = \frac{n_0 c}{2} \frac{2\sqrt{2}}{\pi \psi_w^{1/2}}, \tag{11}
$$

which is 0.974 times smaller than $n_0 c / 2$, the maximal plasma particle flux-density in the fluid picture.[24]

Since $n = n_0 \exp(-\psi)$ we write

$$
\psi = -\ln n_n \quad n_n \equiv \frac{n}{n_0}. \tag{12}
$$

The ratio of the plasma density at the boundary at $z = a$ to its maximal value is

$$
n_n(a) = \exp(-\psi_m) = 0.426, \tag{13}
$$

while in the fluid picture[24] this ratio is 0.5. The maximal plasma flow velocity is

$$
v_{\max} = \frac{\Gamma_{\max}}{n(a)} = \frac{c}{2} \frac{2\sqrt{2}}{\pi \psi_w^{1/2}} \exp(\psi_w) = 1.144c. \tag{14}
$$

We can express the normalized plasma particle flux-density

$$
\Gamma_n \equiv \frac{\Gamma(\psi)}{\Gamma_{\max}} = 2\psi_w^{1/2} F(\psi^{1/2}), \tag{15}
$$

as a function of the plasma density as

$$
\Gamma_n = 2\psi_w^{1/2} F(\sqrt{-\ln n_n}). \tag{16}
$$

This relation between the plasma particle flux-density and the plasma density is independent of the neutral density. In the fluid picture[24] the equivalent relation is $\Gamma_n^2/4 + n_n^2 = n_n$.

The plasma and neutrals are coupled through ionization. We write the ionization rate as

$$
g = \beta(T) nN, \tag{17}
$$

where N is the neutral density. In the next section we describe the neutral dynamics that determines the profile of N.

IV. THE COLLISIONLESS NEUTRAL DYNAMICS

We assume that the neutrals, like the plasma ions, are collisionless, that they move ballistically, and that they are coupled to the plasma only through volume ionization and wall recombination. The neutral dynamics assumed here is identical to that in our previous publications,[21,24] where the plasma was treated within a fluid model. Similar analyses of neutrals that move ballistically[2–4,9,12,14] exhibit neutral depletion similar to that shown here. In Ref. 24 we derived a formalism for the neutrals of a general distribution at the wall. As we did in Ref. 24, we choose to apply the formalism to the following simple case.

We assume that the neutral gas is composed of two monoenergetic counterstreaming beams

$$
f_N(v,z) = \frac{\Gamma_1(z)}{v_a} \delta(v - v_a) + \frac{\Gamma_2(z)}{v_a} \delta(v + v_a). \tag{18}
$$

The neutral density, particle flux-density and velocity are

$$
N(z) = \frac{\Gamma_1(z) + \Gamma_2(z)}{v_a}, \quad \Gamma_N(z) = \Gamma_1(z) - \Gamma_2(z),
$$

$$
v_N(z) = v_a \frac{\Gamma_1(z) - \Gamma_2(z)}{\Gamma_1(z) + \Gamma_2(z)}, \tag{19}
$$

while the neutral pressure is

$$
P_N(z) = \int_{-\infty}^{\infty} dv f_N(v,z) m(v - v_N)^2 = \frac{4m \Gamma_1(z) \Gamma_2(z)}{N(z)}. \tag{20}
$$

From the continuity equations for the two separate beams

$$
\frac{d\Gamma_1}{dz} = -\beta n \frac{\Gamma_1}{v_a}, \quad \frac{d\Gamma_2}{dz} = \beta n \frac{\Gamma_2}{v_a}, \tag{21}
$$

it follows that

$$
\Gamma_1(z) \Gamma_2(z) = \Gamma_0^2 = \text{const}. \tag{22}
$$

The pressure is therefore

$$
P_N(z) = \frac{4m \Gamma_0^2}{N(z)}. \tag{23}
$$

The neutral pressure is inversely proportional to the neutral density.

Let us assume that there is no net mass flow, so that the sum of the neutral particle flux-density and the plasma particle flux-density is zero,

$$\Gamma(z) + \Gamma_1(z) - \Gamma_2(z) = 0. \tag{24}$$

Combining Eqs. (22) and (24) we find that

$$\Gamma_1(z) = -\frac{\Gamma(z)}{2} + \sqrt{\frac{\Gamma^2(z)}{4} + \Gamma_0^2},$$

$$\Gamma_2(z) = \frac{\Gamma(z)}{2} + \sqrt{\frac{\Gamma^2(z)}{4} + \Gamma_0^2}, \tag{25}$$

and that the neutral density is

$$N = \frac{\sqrt{\Gamma^2(z) + 4\Gamma_0^2}}{v_a} = \frac{\Gamma_{max}}{v_a}\sqrt{\Gamma_n^2 + B_{cl}^2} \quad B_{cl} \equiv \frac{2\Gamma_0}{\Gamma_{max}}. \tag{26}$$

We assume further that there are walls located at $z = \pm a$ and that the discharge is symmetric with respect to $z = 0$. We define the neutral depletion as

$$D \equiv \frac{N(a) - N(0)}{N(0)}, \tag{27}$$

which turns out in our case to be

$$D = \sqrt{1 + \frac{1}{B_{cl}^2}} - 1. \tag{28}$$

It is clear from Eq. (26) that

$$\frac{dN(z)}{dz} < 0 \quad -a \leq z < 0; \quad \frac{dN(z)}{dz} > 0 \quad 0 < z \leq a. \tag{29}$$

Thus, these collisionless neutrals that move ballistically between the walls have a lower density away from the walls. This density profile is opposite in its shape to the density profile of the thermalized neutral-gas analyzed in Ref. 22 in which the maximal density is at the center of the discharge. In Ref. 24 we showed that collisionless neutrals that move ballistically between the walls always have a lower density away from the walls, not only the neutral flow composed of two cold beams as in Eq. (18). The source of the difference between the density profiles of collisionless and thermalized neutral-gas is discussed in the next section.

V. DENSITY AND PRESSURE OF THE NEUTRAL GAS

The steady neutral-gas distribution function $f_N(v, z)$ is a solution of the time-independent Boltzmann equation,

$$\frac{d(vf_N)}{dz} = -\beta n f_N. \tag{30}$$

The continuity and the momentum equations for the neutrals, that are obtained from Eq. (30), are

$$\frac{d(NV)}{dz} = -\beta Nn, \quad \frac{d(mNV^2 + p_N)}{dz} = -m\beta NnV. \tag{31}$$

These equations are combined to

$$mNV\frac{dV}{dz} + \frac{dp_N}{dz} = 0. \tag{32}$$

From the momentum equation in this form [Eq. (32)], it follows that if the neutral velocity decreases along the neutral flow (say $dV/dz < 0$ as $NV > 0$), then the neutral-gas pressure should increase along the neutral flow ($dp_N/dz > 0$). For the configuration of a symmetrical discharge between the two walls, in both cases of interest to us, thermalized and collisionless neutrals, the neutrals flow from the wall towards the center of the discharge. The neutral flow-velocity decreases along the neutral flow and vanishes at the center of the discharge. Therefore, the neutral gas-pressure should increase from the wall towards the center of the discharge, becoming maximal at the center.

Let us examine now the profile of the neutral density. In the case that $p_N = p_N(N)$ the neutral density profile is determined by the relation

$$\frac{dp_N}{dz} = \frac{dp_N}{dN}\frac{dN}{dz}. \tag{33}$$

We examine first the case of thermalized neutrals, that was studied in Ref. 22. The neutrals were assumed to satisfy the equation of state

$$p_N = NT_g, \tag{34}$$

where T_g, the gas temperature, was assumed constant. Since

$$\frac{dp_N}{dN} = T_g > 0, \tag{35}$$

the spatial derivative of the neutral density has the same sign as the spatial derivative of the pressure,

$$\frac{dp_N}{dz} \bigg/ \frac{dN}{dz} > 0. \tag{36}$$

Therefore, since the neutral-gas pressure increases along the neutral flow, so does the neutral density. In the configuration of a discharge between two walls the neutral-gas density turns out to be higher away from the wall, what we call a neutral **repletion**.

The second case is of collisionless neutrals analyzed in this paper. The equation of state is Eq. (23),

$$P_N(z) = \frac{4m\Gamma_0^2}{N(z)}. \tag{37}$$

The neutral pressure is inversely proportional to the neutral density and, following Eq. (33)

$$\frac{dp_N}{dN} = -\frac{4m\Gamma_0^2}{N^2} < 0 \Rightarrow \frac{dp_N}{dz} \bigg/ \frac{dN}{dz} < 0. \tag{38}$$

In this case, therefore, since the neutral pressure increases the neutral density decreases along the neutral flow. In the collisionless case the neutral density has its minimum where the neutral pressure has its maximum, at the center of the discharge. Thus, the discharge with collisionless neutrals analyzed here exhibits a neutral **depletion**.

In summary, both for collisionless and thermalized neutrals, the pressure of the neutral-gas is higher at the center

and lower near the walls. The equation of state of the neutrals, the relation between the neutral density and pressure, determines whether there will be an ionization-induced maximal neutral density at the center, called neutral repletion, or, in contrast, a neutral depletion.

VI. NUMERICAL SOLUTIONS

Employing the expressions for the ionization rate Eq. (17) and for the neutral density Eq. (26), we write the continuity equation (1) in a dimensionless form as

$$\frac{d\Gamma_n}{d\xi} = P_n n_n \sqrt{\Gamma_n^2 + B_{cl}^2},$$
(39)

where

$$\xi \equiv \frac{z}{a}, \quad P_n \equiv \frac{\beta a n_0}{v_a}.$$
(40)

Integrating this equation between the center of the discharge and the wall yields a relation between the two parameters P_n and B_{cl},

$$P_n = \int_0^1 \frac{d\Gamma_n}{n_n \sqrt{\Gamma_n^2 + B_{cl}^2}}.$$
(41)

When ionization is small, $B_{cl} \gg 1$, the neutral density is approximately constant and Eq. (39) is approximated as

$$\frac{d\Gamma_n}{d\xi} = P_n B_{cl} n_n.$$
(42)

Employing the relations $d\Gamma_n/d\xi = (d\Gamma_n/d\psi)(d\psi/d\xi)$ and $d\Gamma_n/d\psi = 2\psi_w^{1/2}[-F(\psi^{1/2}) + 0.5\psi^{-1/2}]$ [from Eq. (9)], we obtain the equation

$$\frac{P_n B_{cl}}{2\psi_w^{1/2}}\xi = \frac{\sqrt{2}\pi\beta a N_0}{c}\xi$$
$$= \int_0^{\psi^{1/2}} ds[1 - 2sF(s)]\exp(s^2), \quad N_0 \equiv \frac{2\Gamma_0}{v_a}. \quad (43)$$

Here N_0 is the uniform neutral density. We write the integral in this case of a uniform neutral density in two equivalent forms,

$$\frac{\pi\beta a N_0}{2\sqrt{2}c}\xi = \frac{\sqrt{\pi}}{2}\left(\frac{1}{2} - \psi\right)\text{erf } i(\psi^{1/2}) + \frac{\psi^{1/2}}{2}\exp(\psi)$$
$$= \left[\left(\frac{1}{2} - \psi\right)F(\psi^{1/2}) + \frac{\psi^{1/2}}{2}\right]\exp(\psi). \quad (44)$$

Substituting the boundary condition $\xi(\psi = \psi_w) = 1$, we obtain from the vanishing of the square brackets in Eq. (9) the relation

$$\frac{\pi\beta a N_0}{2\sqrt{2}c} = \frac{\exp(\psi_w)}{4\psi_w^{1/2}} = 0.6355.$$
(45)

For the collisionless plasma interacting with a uniform density neutral the kinetic treatment results here in $\beta a N_0/c = 0.5721$ while the cold-fluid treatment in Ref. 24 results in $\beta a N_0/c = (\pi - 2)/2 = 0.5708$. In both treatments the parameter
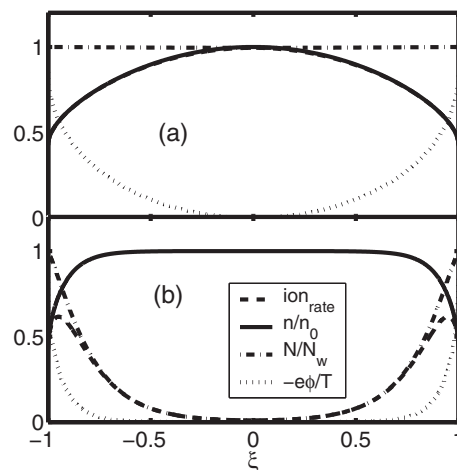

FIG. 1. The normalized plasma density $n_n = n/n_0$, normalized neutral density N/N_w, normalized ionization rate $n_n N/N_w$, and normalized electric potential ψ for (a) $B_{cl}^2 = 100$, (b) $B_{cl}^2 = 0.0001$.

$\beta(T)/c(T)$ and therefore also the electron temperature are determined by the Paschen parameter aN_0.

Let us turn now to the case of a high neutral depletion. When neutral depletion is high, however, $B_{cl} \ll 1$, most of the contribution is from the region in which $n_n \cong 1$. Equation (41) is then approximated as[2,24]

$$P_n = \int_0^1 \frac{d\Gamma_n}{\sqrt{\Gamma_n^2 + B_{cl}^2}} \cong \int_0^1 \frac{d\Gamma_n}{\sqrt{\Gamma_n^2 + B_{cl}^2}}$$
$$= \ln\left(\frac{1 + \sqrt{B_{cl}^2 + 1}}{B_{cl}}\right) \cong \ln\left(\frac{2}{B_{cl}}\right). \quad (46)$$

We write this approximate relation as $B_{cl} \cong 2\exp(-P_n)$ and the neutral depletion is then expressed as

$$D \cong \frac{1}{B_{cl}} = 0.5\exp(P_n) \quad B_{cl} \ll 1.$$
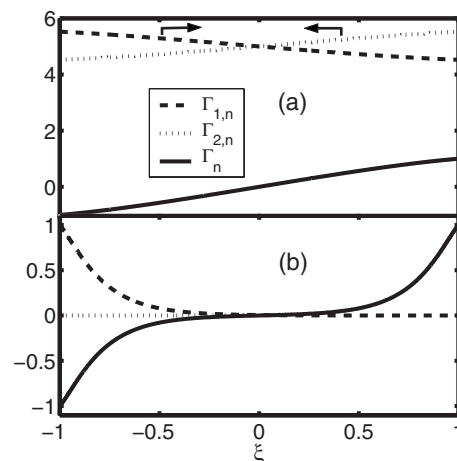(47)

In the general case we solve numerically

FIG. 2. The normalized plasma flow Γ_n and neutral flows $\Gamma_{1,n}$ and $\Gamma_{2,n}$ for (a) $B_{cl}^2 = 100$, (b) $B_{cl}^2 = 0.0001$.
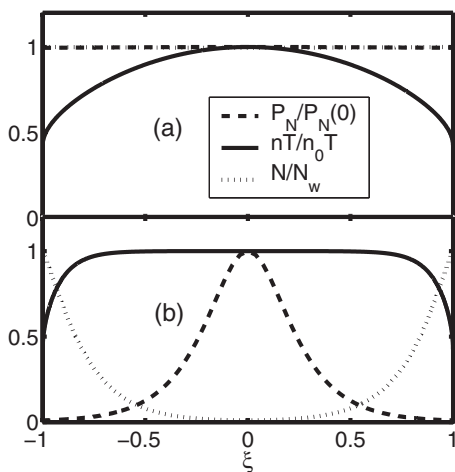
FIG. 3. The normalized plasma pressure $nT/nT_0 = n_n$, the normalized neutral pressure $P_N/P_N(0) = 1/\sqrt{1 + \Gamma_n^2/B_{cl}^2}$, and the normalized neutral density N/N_w for (a) $B_{cl}^2 = 100$, (b) $B_{cl}^2 = 0.0001$.

$$P_n \xi = \int_0^{\psi^{1/2}} ds [1 - 2sF(s)] \frac{\exp(s^2)}{\sqrt{F^2 + B_n^2}} \quad B_n^2 \equiv \frac{B_{cl}^2}{4\psi_w}. \quad (48)$$

The boundary condition $\xi(\psi = \psi_w) = 1$ results in

$$P_n = \int_0^{\psi_w^{1/2}} ds (1 - 2sF) \frac{\exp(s^2)}{\sqrt{F^2 + B_n^2}}. \quad (49)$$

Equation (48) then becomes

$$\xi = \frac{\int_0^{\psi^{1/2}} ds \exp(s^2)(1 - 2sF)(F^2 + B_n^2)^{-1/2}}{\int_0^{\psi_w^{1/2}} ds \exp(s^2)(1 - 2sF)(F^2 + B_n^2)^{-1/2}}. \quad (50)$$

For a specified B_{cl}^2 we find P_n and $\xi(\psi)$. The normalized plasma and neutral fluxes are then

$$\Gamma_n = 2\psi_w^{1/2} F(\psi^{1/2}), \quad \Gamma_{1,n} \equiv \frac{\Gamma_1}{\Gamma_{max}} = -\frac{\Gamma_n}{2} + \frac{1}{2}\sqrt{\Gamma_n^2 + B_{cl}^2},$$

$$(51)$$

$$\Gamma_{2,n} \equiv \frac{\Gamma_2}{\Gamma_{max}} = \frac{\Gamma_n}{2} + \frac{1}{2}\sqrt{\Gamma_n^2 + B_{cl}^2}.$$

The neutral density, normalized with respect to the neutral density at the wall N_w, is

$$\frac{N}{N_w} = \sqrt{\frac{\Gamma_n^2 + B_{cl}^2}{1 + B_{cl}^2}}. \quad (52)$$

In Figs. 1–3 we present the profiles of the plasma and neutral flow variables for two cases, low neutral depletion ($B_{cl}^2 = 100$) and high neutral depletion ($B_{cl}^2 = 0.0001$). In Fig. 1 are shown the normalized plasma density $n_n = n/n_0$, the normalized neutral density N/N_w, the normalized ionization rate $n_n N/N_w$, and the normalized electric potential ψ. It is seen in the figure that the plasma density and the electric potential profiles are much flatter when neutral depletion is high. The normalized ionization rate almost coincides with the plasma density when neutral depletion is low and is maximal at the center of the discharge. When neutral depletion is high the ionization rate is very low at the center and is maximal near the walls.

Figure 2 shows the normalized plasma flow Γ_n and neutral flows $\Gamma_{1,n}$ and $\Gamma_{2,n}$ for low and high neutral depletion for the two values of B_{cl}^2 mentioned above. When neutral depletion is low the two opposite neutral fluxes are symmetrical with respect to $z = 0$. When neutral depletion is high $\Gamma_n \cong -\Gamma_{1,n}$ for $z < 0$ and $\Gamma_n \cong \Gamma_{2,n}$ for $z > 0$.

Figure 3 shows the profiles of the normalized plasma pressure $nT/nT_0 = n_n$, the normalized neutral pressure $P_N/P_N(0) = 1/\sqrt{1 + \Gamma_n^2/B_{cl}^2}$, and the normalized neutral density N/N_w. It is seen in the figure that the neutral pressure is approximately constant when neutral depletion is low, and is peaked at the center when neutral depletion is high, being inversely proportional to the neutral density.

VII. SUMMARY

In this paper we reviewed our recent theoretical studies of neutral depletion and repletion that follow strong ionization in gas discharges. We showed that what determines which of the two processes, depletion or repletion, will occur is the relation between density and pressure in the neutral gas. For collisionless neutrals the pressure is inversely proportional to the density, resulting in neutral depletion, while when neutrals are thermalized so that their pressure increases with their density, an unexpected neutral repletion is induced. We have performed here again the analysis of collisionless neutrals coupled to plasma through ionization and wall recombination only, with a kinetic model for the plasma ions.

ACKNOWLEDGMENTS

[1] J. E. Allen and P. C. Thonemann, Proc. Phys. Soc. London, Sect. B **67**, 768 (1954).
[2] A. Caruso and A. Cavaliere, Br. J. Appl. Phys. **15**, 1021 (1964).
[3] P. C. Stangeby and J. E. Allen, J. Phys. A **4**, 108 (1971); J. Phys. D **6**, 1373 (1973).
[4] H. B. Valentini, J. Phys. D **17**, 931 (1984).
[5] R. W. Boswell and K. Porteous, Appl. Phys. Lett. **50**, 1130 (1987).
[6] I. D. Sudit and F. F. Chen, Plasma Sources Sci. Technol. **5**, 43 (1996).
[7] J. Gilland, J. R. Breun, and N. Hershkowitz, Plasma Sources Sci. Technol. **7**, 416 (1998).
[8] A. W. Degeling, T. E. Sheridan, and R. W. Boswell, Phys. Plasmas **6**, 1641 (1999); **6**, 3664 (1999).
[9] S. Yun, K. Taylor, and G. R. Tynan, Phys. Plasmas **7**, 3448 (2000).
[10] E. J. Tonnis and D. B. Graves, J. Vac. Sci. Technol. A **20**, 1787 (2002).
[11] H. Abada, P. Chabert, J. P. Booth, J. Robiche, and G. Gartry, J. Appl. Phys. **92**, 4223 (2002).
[12] C. Watts and J. Hanna, Phys. Plasmas **11**, 1358 (2004).
[13] B. Clarenbach, M. Krämer, and B. Lorenz, J. Phys. D **40**, 5117 (2007).
[14] J. E. Maggs, T. A. Carter, and R. J. Taylor, Phys. Plasmas **14**, 052507 (2007).
[15] M. Shimada, G. R. Tynan, and R. Cattolica, Plasma Sources Sci. Technol. **16**, 193 (2007).
[16] A. M. Keesee and E. E. Scime, Rev. Sci. Instrum. **77**, 10F304 (2006); Plasma Sources Sci. Technol. **16**, 742 (2007).

[17] A. Aanesland, L. Liard, G. Leray, J. Jolly, and P. Chabert, Appl. Phys. Lett. **91**, 121502 (2007).

[18] D. O'Connell, T. Gans, D. L. Crintea, U. Czarnetzki, and N. Sadeghi, J. Phys. D **41**, 035208 (2008).

[19] A. Fruchtman, G. Makrinich, P. Chabert, and J. M. Rax, Phys. Rev. Lett. **95**, 115002 (2005).

[20] A. Fruchtman, "The effect of the magnetic field profile on the plume and on the plasma flow in the Hall thruster," in *Proceedings of the 29th International Electric Propulsion Conference*, Princeton, NJ, 2005 (Electric Rocket Propulsion Society, Cleveland, OH, 2005), IEPC Paper No. 2005-068.

[21] A. Fruchtman, "Neutral depletion and pressure balance in plasma," *33rd European Physical Society Conference on Plasma Physics*, Rome, Italy, 2006, edited by F. De Marco and G. Vlad, European Conference Abstracts, ISBN 2-914771-40-1, Vol. 30I, Paper No. D-5.013.

[22] J.-L. Raimbault, L. Liard, J.-M. Rax, P. Chabert, A. Fruchtman, and G. Makrinich, Phys. Plasmas **14**, 013503 (2007).

[23] L. Liard, J.-L. Raimbault, J.-M. Rax, and P. Chabert, J. Phys. D **40**, 5192 (2007).

[24] A. Fruchtman, "Neutral depletion in a collisionless plasmas," IEEE Trans. Plasma Sci. (to be published).

[25] A. Fruchtman, "Energizing and depletion of neutrals by a collisional plasma," Plasma Sources Sci. Technol. (submitted).

[26] W. Schottky, Phys. Z. **25**, 635 (1924).

[27] J. R. Forrest and R. N. Franklin, Br. J. Appl. Phys. **17**, 1569 (1966); R. N. Franklin, *Plasma Phenomena in Gas Discharges* (Clarendon, Oxford, 1976).

[28] V. A. Godyak, *Soviet Radio Frequency Discharge Research* (Delphic Associates, Falls Church, 1986).

[29] M. A. Lieberman and A. J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing* (Wiley, New York, 1994).

[30] A. Fruchtman, G. Makrinich, and J. Ashkenazy, Plasma Sources Sci. Technol. **14**, 152 (2005).

[31] N. Sternberg, V. Godyak, and D. Hoffman, Phys. Plasmas **13**, 063511 (2006).

[32] C. Charles, Plasma Sources Sci. Technol. **16**, R1 (2007).

[33] F. F. Chang-Diaz, Sci. Am. **283**, 90 (2000).

[34] W. M. Manheimer and R. F. Fernsler, IEEE Trans. Plasma Sci. **29**, 75 (2001).

[35] A. Fruchtman, Phys. Rev. Lett. **96**, 065002 (2006).

[36] A. Dunaevsky, Y. Raitses, and N. J. Fisch, Appl. Phys. Lett. **88**, 251502 (2006).

[37] F. F. Chen, "Permanent magnet helicon source for ion propulsion," IEEE Trans. Plasma Sci. (submitted).

[38] R. Winglee, T. Ziemba, L. Giersch, J. Prager, J. Carscadden, and B. R. Roberson, Phys. Plasmas **14**, 063501 (2007).

[39] M. Manente, J. Carlsson, I. Musso, C. Bramanti, D. Pavarin, and F. Angrilli, "Numerical simulation of the Helicon Double Layer Thruster Concept," AIAA Paper No. 2007-5312 (2007).

[40] O. Batishchev, N. Sinenian, M. Celik, and M. Martinez-Sanchez, "Development of the minihelicon thruster at MIT," in *Proceedings of the 30th International Electric Propulsion Conference*, Florence, Italy, 2007 (Electric Rocket Propulsion Society, Cleveland, OH, 2007), IEPC Paper No. 2007-355.

[41] D. Palmer, C. Akinli, L. Williams, and M. L. R. Walker, "Characterization of an annular helicon plasma source," in *Proceedings of the 30th International Electric Propulsion Conference*, Florence, Italy, 2007 (Electric Rocket Propulsion Society, Cleveland, OH, 2007), IEPC Paper No. 2007-202.

[42] L. Tonks and I. Langmuir, Phys. Rev. **34**, 876 (1929).

[43] E. R. Harrison and W. B. Thompson, Proc. Phys. Soc. London **74**, 145 (1959).

[44] S. Self and H. N. Ewald, Phys. Fluids **9**, 2486 (1966).

[45] G. S. Kino and E. K. Shaw, Phys. Fluids **9**, 587 (1966).